

HEAT TRANSFER OUT OF ACLINIC CIRCULAR CYLINDER BY MEANS OF AN ELECTRICALLY CONDUCTING LIQUID WITH VISCOUS DISSIPATION, JOULE HEATING AND STRESS WORK

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Abstract— Under consideration of stress work, Joule heating and viscous dissipation, heat transfer due to combined effect of conduction and natural convection out of a thermally uniform circular cylinder over an electrically conducting liquid is investigated. The equations leading the flow and related frontier conditions are formed dimensionless by a group of dimensionless factors. The equations are quantitatively solved using the implicit finite difference method (IFDM). The Prandtl number, the Joule heating factor, the viscous dissipation factor, and the stress work factor are used to numerically calculate the temperature and velocity within the boundary layer, while the skin friction of the surface and the heat transfer in terms of Nusselt number along the surface are observed.

Keywords— Combine natural convection, MHD, aclinic cylinder, viscous dissipation, stress work, Finite difference method, Joule heating.

I. INTRODUCTION

Conjugate heat transfer is important in a variety of applications because of the interaction of conduction within the solid body and convection movement to the adjacent fluid on the solid surface. Indeed, the convection of the surrounding fluid has a significant influence on conduction within the solid wall. Furthermore, planetary and stellar magnetospheres, chemical engineering, aeronautics, and electronics have all seen applications of magnetohydrodynamic (MHD) naturalistic convective flow. So it is very crucial to analyse the simultaneous effect of fluid convection and solid conduction in presense of magnetohydrodynamics.

Many scholars used a variety of numerical techniques to examine laminar natural convection and conjugate free convection in two dimensions on a steep surface and under various surface frontier conditions, from a aclinic cylinder. The naturalistic convective flow beneath boundary layer upon a thermal symmetrical cylinder with a fixed temperature flux was examined by Merkin and Pop [1,2]. They also studied conjugate free convection from vertical surface. They solve the equations governing the problem using finite difference method and found that Blasius expansion method is better at estimating temperature and Gortler-type expansion is better at calculating velocity profiles. Using an elliptic numerical procedure, Kuehn and Goldstein [3] explained the continuity, momentum, and energy equations that resulted from laminar naturalistic convective movement from cylinders. Wang et al. [4], using spline fractional step, examined numerical calculation of laminar naturalistic convective flow because of a heated aclinic cylinder under various surface conditions.

Natural convection conjugate problems had been investigated by Gdalevich and Fertman [5] in 1977. Miyamoto et al. [6] looked at how heat conduction play pivotal role on heat transfer by free convection in a steep flat plate. Two expansions—the first a regular series and the second an asymptotic expansion—were used by Pozzi and Lupo [7] to study the interaction of naturalistic convection to the adjacent fluid and conduction along a heated flat plate. Kimura and Pop [8] revealed the properties of a steady, laminate free convective stream out of a circular aclinic cylinder while taking thermal conduction and a heated core region into account.

A significant amount of research has also been done to find out the outcome of a transverse magnetic field applied on the fluid flux and heat moving properties in various geometries for electrically conducting fluids. For instance, Wilks et al. [9] investigated magnetohydrodynamic free convection over a slightly bounded steep plate in a hefty diagonal field. The combined effects of viscosity and Joule dissipation energy on magnetohydrodynamic naturalistic convective flow through a semi-unbounded isothermal steep plate under a criss-cross magnetic region was determined by Takhar and Soundalgecker [10]. They discovered that as the values of the magnetic region strength increase, the outcomes of viscous dissipation energy and energy produced due to Joule heating phenomena become more dominant. Hossain [11] looked into the outcomes of viscous energy and energy produced by ohmic heating effect on MHD naturalistic convective flow. The impact of an axial magnetic region on mixed convection flow



from a aclinic cylinder was examined by Aldoss et al. [12]. They found that an increment in the magnetic strength capacity causes heat movement, share stress, and velocity to decrease. Chamkha [13] investigated the hydromagnetic physical convection that originated from an bended isothermal surface near a thermally stratified porous material. Yih [14] studied magnetohydrodynamnic convection flow created by external forced along a non-uniform temperature distributed wedge with stress work, electromagnetic and viscous energy. El-Amin [15] investigated first- and second-order effects on forced convection flow in aclinic cylinders created by non-Darcy porous media, viscous and Joule-removing solid matrices. Mollah and others. [16] investigated the naturally occurring convection flow from a uniform temperature distributed aclinic circular cylinder considering heat propagation. The flow across a uniform temperature distributed aclinic circular cylinder considering viscosity as a function of temperature was investigated by Ahmad et al. [17]. Joshi and Gebert [18] investigated how viscous energy and shear stress affected natural convection flow. They discovered that the pressure work has a much greater impact on heat transfer than viscous dissipation. Pantokratoras [19] looked into the impact of stress work as well as viscous energydriven natural convection along a steep uniform temperature distributed plate. The impact of Hartmann, Joule heating, Brinkmann, and Revnolds numbers on force convection flow in parallel-plate microchannels on flow and temperature was recently investigated by Pordanjani et al. [20]. They used control volume finite difference methods to discover the discrete equations' numerical solutions.

In the current study, the effects of energy on account of viscous dissipation, energy from Joule or resistive or ohmic heating and stress work are taken into account while analysing conjugate naturalistic convective heat transfer flow across a fluid which is electrical conductive. The non-dimensional leading boundary layer equations are numerically solved using IFDM and Keller box methodology [21, 22]. The velocity profiles, temperature profiles, coefficient of skin friction to the surface, and heat transfer rate are all graphically depicted for a range of factor values. The precise derivation of the equations leading the flow field and heat transfer, as well as the method of solution and results, are covered in the subsequent sections.

II. MATHEMATICAL FORMATION

Take into account a uniform temperature distributed, aclinic, circular cylinder with radius *a* that is submerged in an electrically conducting fluid with constant

temperature T_{∞} (see Fig. 1). The liquid is incompressible and viscous. The cylinder has a core area that is heated to a certain temperature T_b , and the typical distance between the inside and outside surfaces is *b*. The temperature of the inner region T_b is higher than the ambient fluid temperature T_{∞} .

A hefty constant magnetic field of capacity "B₀" is acting normally towards the surface of the cylinder. The persuaded magnetic field is disregarded, and the other fluid features are taken to be immutable. The \overline{y} -axis is calculated upright to the exterior, and the \overline{x} -axis is drawn along the cylinder's edge as measured from its inferior stagnation point.



Fig. 1: Dimensions of the issue

The equations regulating this natural convection flow within boundary layer while "the body force term of the momentum equation uses the Boussinesq approximation and is expressed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\overline{u}}{\partial \overline{x}} + \frac{\overline{v}}{\partial \overline{y}} = v \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + g \beta \left(T_f - T_\infty\right) \sin\left(\frac{\overline{x}}{a}\right) - \frac{\sigma B_0^2 \overline{u}}{\rho}$$
(2)

$$\frac{\overline{u}}{\partial \overline{x}}^{\partial T_{f}} + \overline{v}\frac{\partial T_{f}}{\partial \overline{y}} = \frac{\kappa}{\rho c_{p}}\frac{\partial^{2} T_{f}}{\partial \overline{y}^{2}} + \frac{\nu}{c_{p}}\left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)^{2} + \frac{\sigma B_{0}^{2} \overline{u}^{2}}{\rho c_{p}} + \frac{T\beta \overline{u}}{\rho c_{p}}\frac{\partial p}{\partial \overline{x}}$$
(3)

The following boundary conditions are suggested by the system's physical configuration [8, 23]:

$$\begin{split} & \overline{u} = \overline{v} = 0, \ T_f = T(\overline{x}, 0), \ \frac{\partial T_f}{\partial \overline{y}} = \frac{\kappa_s}{b\kappa_f} \left(T_f - T_b \right) \ on \ \overline{y} = 0, \ x > 0 \\ & \overline{u} \to 0, \ T_f \to T_{\infty} \ as \ \overline{y} \to \infty, \ \overline{x} > 0 \end{split}$$
(4)

By employing the Grashof number, $Gr = [g\beta a^3(T_b - T_{\infty})]/v^2$ that is assumed to be large, and the non-dimensional quantities stated as:

$$x = \frac{\bar{x}}{a}, \ y = \frac{\bar{y}}{a}Gr^{1/4}, \ u = \frac{\bar{u}a}{v}Gr^{-1/2}, \ v = \frac{\bar{v}a}{v}Gr^{-1/4}, \ \theta = \frac{T_f - T_{\infty}}{T_b - T_{\infty}}$$
(5)

where θ is the heat without dimension. The following are the governing equations (1) through (3) in their nondimensional forms:



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + Mu = \frac{\partial^2 u}{\partial y^2} + \theta \sin x$$
(7)

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2} + N\left(\frac{\partial u}{\partial y}\right)^2 + Ju^2 - \varepsilon t_r u - \varepsilon u\theta \qquad (8)$$

Where, $M = (\sigma a^2 B_0^2) / (\nu \rho G r^{1/2})$ is the magnetic factoris, Prandtl number is $Pr = (\mu c_p) / \kappa$, the viscous dissipation factor is $N = (\nu^2 G r) / \{a^2 c_p (T_b - T_\infty)\}$, the Joule heating factor is $J = (\sigma \nu B_0^2 G r^{1/2}) / \{\rho c_p (T_b - T_\infty)\}$, the Stress work factor is $\varepsilon = (g\beta a) / (c_p)$, and $t_r = T_\infty / (T_b - T_\infty)$ is the Temperature ratio."

The following dimensionless form can be used to express boundary condition (4):

$$u = v = 0, \ \theta - 1 = \chi \frac{\partial \theta}{\partial y}, \ on \ y = 0, \ x > 0$$

$$u \to 0, \ \theta \to 0 \ as \ y \to \infty, \ x > 0$$
(9)

In equation (9), the factor for conjugate conduction is $\chi = (b\kappa_f G r^{1/4})/(a\kappa_s)$. The size of the Prandtl number Pr determines the current issue. Pr as well as other factors like M, χ , N, T, ε and t_r . The value of t_r is considered 1.0 theoritically for simplicity". The factor for conjugate conduction χ depends on the Grashof number Gr, and the ratios κ_f / κ_s and b/a. Both the ratios are less than 1, but the value of the Grashop number Gr is very high for the free convection. As a result, χ has a value more than 0. When $\chi = 0$, the current analysis will focus only on the free convection.

We assume the following transformations in order to solve (6)-(8) subject to boundary value found in equation (9).

$$\theta = \theta(x, y), \ \psi = x f(x, y)$$
 (10)

where ψ is the stream function and θ is the dimensionless temperature, the stream function is typically defined as:

$$u = \partial \psi / \partial y$$
 and $v = -\partial \psi / \partial x$ (11)

The dimensionless equations (7) and (8) take on new forms when (11) is substituted into the equations (6) through (9):

$$f''' + ff'' - f'^{2} - Mf' + \theta \frac{\sin x}{x} = x \left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right)$$
(12)

$$\frac{1}{\Pr}\theta'' + f\theta' + Nx^2 f''^2 + Jx^2 f'^2 - x\varepsilon t, f' - x\varepsilon f'\theta = x \left(f' \frac{\partial \theta}{\partial x} - \theta' \frac{\partial f}{\partial x} \right)$$
(13)

The new form of the boundary conditions specified in equation (9) that relate to this is:

$$f = f' = 0, \ \theta - 1 = \chi \frac{\partial \theta}{\partial y} \ at \ y = 0, \ x > 0$$

$$f' \to 0, \ \theta \to 0 \ as \ y \to \infty, \ x > 0$$
 (14)

In the aforementioned equations, primes only denote differentiation with regard to y. The governing equations (12) and (13) are numerically solved using the implicit finite difference approach utilising the Keller box technique [21, 22] while taking into account the boundary conditions stated in equation (14). Fundamental physical quantities such as the heat transfer rate and shear stress can be written as [16] using the Nusselt number and skin friction coefficient, respectively.

$$Nu = (aq_w) / \{\kappa(T_w - T_w)\} \text{ and } C_f = \tau_w / (\rho U_w^2)$$
(15)
Where $\tau_w = \mu \left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)_{\overline{y}=0}$ and $q_w = -\kappa \left(\frac{\partial T}{\partial \overline{y}}\right)_{\overline{y}=0}$

Using the equations (5) and (14), we have

$$C_f Gr^{1/4} = x f''(x,0), \ Nu Gr^{-1/4} = -\theta(x,0)$$
 (16)

The following relationships can be used to derive the numerical values of temperature distributions and velocity profiles:

$$\theta = \theta(x, y), \, u = f'(x, y) \tag{17}$$

III. RESULTS AND DISCUSSION

The current problem's goal is to numerically solve the leading equations of the fluid flow and energy transportation around an equilibrium aclinic circular cylinder that conducts electricity, as well as the rate of energy transportation from the core region to the connected fluid. For the simulation, the Prandtl numbers are 0.733, 1.0, 1.44, and 1.63, which are fitted to hydrogen, steam, water, and glycerin, respectively.

The following values are used for the other factors: the magnetic factor M=0.1, the conjugate conduction factor χ =1.0, the temperature ratio t_r =1.0, the viscous dissipation factor N=0.01-1.00, the Joule heating factor J=0.01-1.00 and the stress work factor \mathcal{E} =0.01-0.20.

Table 1: Agreement of current quantitative values with those found in Molla et al. [17] and Merkin [1] for



different x values when $Pr = 1.0 M = 0.0 \chi = 0.0 J = 0.0$ N = 0.0 and $\varepsilon = 0.0$ are used.

	$Nu Gr^{-1/4} = -\theta'(x,0)$			$C_f Gr^{1/4} = x f''(x,0)$		
x	Merkin [1]	Molla et al. [16]	Present	Merkin [1]	Molla et al. [16]	Present
0.0	0.4214	0.4214	0.4215	0.0000	0.0000	0.0000
π/3	0.4007	0.4005	0.4005	0.7558	0.7539	0.7527
2π/3	0.3364	0.3355	0.3356	0.9756	0.9696	0.9677
π	0.1945	0.1917	0.1911	0.3391	0.3264	0.3238

Table 1 compares the local Nusselt number and local skin friction coefficient determined in the current study with $Pr = 1.0 \ \chi = 0.0 \ M = 0.0 \ J = 0.0 \ N = 0.0$ and $\varepsilon = 0.0$ and and determined by Molla et al.[16] and Merkin [1]. Table 1 makes it abundantly evident that the three results are in perfect agreement with one another.



Fig.2: (a) Velocity and (b) Temperature profiles within the boundary layer against *y* for multiple values of *Pr* while $N = 0.01 \ J = 0.01 \ \mathcal{E} = 0.1 \ M = 0.1$ and $\chi = 1.0$.



Fig.3: (a) Shear stress in terms of the coefficient of skin friction and (b) Heat transfer rate in form of Nusselt number along x for multiple values of Pr with N = 0.01 J = 0.01 $\mathcal{E} = 0.1$ M = 0.1 and $\chi = 1.0$.

Fig.2 represents the dimensionless velocity and temperature for multiple values of Prandtl number and the shear stress in form of the coefficient of skin friction and the rate of heat transfer in terms of Nusselt number for multiple values of Prandtl number are depicted in fig.3 while $N = 0.01 \ \mathcal{L} = 0.01 \ \mathcal{L} = 0.1 \ \mathcal{M} = 0.1 \ \text{and} \ \chi = 1.0$.

The Prandtl number is defined as the ratio of thermal force to viscous force. As a result, as Pr increases, the fluid's viscosity rises and its thermal action falls. As a result, as shown in figs. 2(a) and (b), the fluid's velocity and temperature should decrease as the Prandtl number increases. As shown in fig. 3(a), increasing the Prandtl number causes a decrease in velocity, which results in a decrease in the skin friction coefficient. Figure 3(b)

shows that as the Prandtl number increases, so does the rate of heat transmission.

Figure 4 shows the impacts of the factor *J* on velocity profiles and temperature distributions, whereas Figure 5 shows the effects of this factor on the coefficients of the skin friction and Nusselt numbers, respectively, with $N = 0.01 \ \varepsilon = 0.1 \ M = 0.1 \ \chi = 1.0$ and Pr = 1.0.



Fig.4: (a) Velocity and (b) Temperature profiles within the boundary layer against *y* for multiple values of *J* while $N = 0.01 \ \mathcal{E} = 0.1 \ M = 0.1 \ \chi = 1.0$ and Pr = 1.0.



Fig.5: (a) Shear stress in terms of the coefficient of skin friction and (b) Heat transfer rate in form of Nusselt number along x for multiple values of J with $N = 0.01 \ \varepsilon = 0.1$ $M = 0.1 \ \chi = 1.0$ and Pr = 1.0.

Electrical resistance and magnetic-field intensity are both components of the Joule heating factor. It converted electrical energy into heat energy due to the presence of electrical resistance. As a result, the temperature of the adjacent fluid rises as J increases, as shown in figure 4(b). Heat convection in the boundary layer area accelerates as thermal energy increases, enhancing fluid motion in the end, as shown in figure 4(a). As the temperature of the boundary layer area rises with increasing Joule heating factor, the temperature difference between the inner and outer parts of the cylinder decreases, and thus heat transfer decreases, as shown in figure 5(b). Figure 5(a) also shows that as the Joule heating factor J increases, so does shear stress.

Figures 7(a), 7(b), 6(a) and 6(b) shows skin friction coefficient, the rate of heat transfer, the velocity and the, temperature for multiple numbers of stress work factor, respectively with $N = 0.1 J = 0.01 M = 0.1 \chi = 1.0$ and Pr = 1.0





Fig.6: (a) Velocity and (b) Temperature profiles within the boundary layer against y for multiple values of \mathcal{E} while N = 0.1, J = 0.01, M = 0.1, $\chi = 1.0$ and Pr = 1.0.



Fig.7: (a) Shear stress in terms of the coefficient of skin friction and (b) Heat transfer rate in form of Nusselt number along x for multiple values of \mathcal{E} with N = 0.01 J = 0.01 M = 0.1 $\chi = 1.0$ and Pr = 1.0.

When the stress work factor, which contains the gravitational force g, is increased, the speed of the fluid flow decreases, as shown by the plot in fig.6(a). As shown in fig.7(a), the reduced velocity slows fluid flow, which reduces shear stress at the wall. However, based on fig.5(b), it is possible to conclude that as the stress work factor increases, the temperature within the boundary layer decreases. As shown in fig.7(b), the heat transfer rate gradually increases as a result of the reduced temperature in the boundary layer area for growing, which reduced the temperature difference between the boundary layer area and the inner part of the cylinder.



Fig.8: (a) Velocity and (b) Temperature profiles within the boundary layer against *y* for multiple values of *N* while $J = 0.01 \ \varepsilon = 0.1 \ M = 0.1 \ \chi = 1.0$ and Pr = 1.0.



Fig.9: (a) Shear stress in terms of the coefficient of skin friction and (b) Heat transfer rate in form of Nusselt number

along x for multiple values of N with J = 0.01 $\mathcal{E} = 0.1$ M = 0.1 $\chi = 1.0$ and Pr = 1.0.

Figures 8(a) and 8(b) show how the velocity and temperature change, respectively as a function of y for multiple values of the viscous dissipation factor N when J = 0.01 $\varepsilon = 0.1$ M = 0.1 $\chi = 1.0$ and Pr = 1.0. Figure 8(a) shows that a modest increase in velocity is correlated with a modest increase in factor N. Its behavior resembles the temperature profile depicted in Figure 8. (b). It suggests that the viscous dissipation raises temperature, which therefore raises velocity. Figures 9(a) and 9(b) show how the factor N affects the shear stress and the local heat transfer rate, respectively while $J = 0.01 \ \varepsilon = 0.1$ M = 0.1 $\chi = 1.0$ and Pr = 1.0. As can be observed the skin friction factor rises as the viscous factor rises. This is expected given that raising N causes the fluid velocity within the boundary layer to increase, which eventually raises the skin friction factor (figure 9(a)). The influence of this factor N results in a reduction in the heat transfer, as seen in figure 9(b).

IV. CONCLUSION

Heat transfer associated with conjugate free convection over electrically conducting fluid out of a uniform temperature distributed aclinic circular cylinder. The effects of joule heating, stress work, and viscous dissipation are investigated. The velocity and temperature of the fluid in the boundary layer area increase as the energy due to viscous dissipation and Joule heating factors increase. However, they are decreasing as the Prandtl number and the stress work factor increase. Furthermore, as the Prandtl number and stress work factor increase, the shear stress in the form of skin friction coefficient along the vertical cross-section of the cylinder on the surface decreases, whereas it increases as the viscous dissipation and Joule heating factors increase. Furthermore, as the viscous dissipation and Joule heating factors increase, so does the heat transfer rate along the surface, while it increases as the stress work factor and Prandtl number increase.

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